Lift! More LIFT!

The American Helicopter Society's 2010 Request for Proposal

Rensselaer Polytechnic Institute Team for the Undergraduate Category: Gun-Smash and the Atlas

Team Members

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Executive Summary

Many ideas went into the initial concept development of our loading system before it was possible to hone in on the final design that would be used in the project. The initial design called for a way to separate the two helicopter system in a safe and effective way. This brought about the concept of a ring or a cross made of I-Beam structures as opposing designs. The ring was initially favored due to the way it was able to distribute forces evenly, but was found to be cumbersome and was considered overkill. This left the rigid cross design, which was made up of two beams that would have wire connections at each endpoint. This concept was simplistic and used far less material than the competing ring design. Since it would be used to carry payload, it was therefore named, "The Atlas." Another concept that was fine-tuned during the design phases of this project was how the cables were going to be attached to the payload. The original plan was to attach cables to the lifting structure, and then from the lifting structure to the payload. The alternate idea was to run the cables straight through the lifting structure directly to the payload, which would use the structure as a cable stay system instead of a lifting system. This would save greatly on weight and even remove a weak point where the connections would be made to the lifting structure.

Once the shape of the system was decided upon and the way the cables were going to be attached was figured out, the cross section design needed to be optimized for buckling and stress. The original plan was to use an I-beam cross section to save on weight while still having a shape that could handle a lot of stress and buckling load. Another idea was to use a thin-walled pipe cross section for the rigid structure. The pipe was selected due to how evenly it could distribute the forces that were applied to it, while the I-beam does not act in a uniform manner regardless of the side inspected. The cable that would hold the entire system is a very important factor in the success of the lifting system. It was essential to find a cable that was strong enough to do the job but that would be as light as possible, as it was crucial not to use too much lifting power getting the system aloft. The cable selected was a 1.125 inch diameter steel rope.

When the structures shape, cross section, and cable selection was decided, it was imperative that the net lifting power usage was under twenty five percent of the total lifting power of the tandem system. If this was not the case, the system would not meet the expected requirements. This concept weighed heavily on every factor that was determined throughout the design process.

It is crucial to have a safe backup plan if there is a catastrophic failure of the system developed. The structure that has been developed has an emergency ballistic parachute that is an option available on the system. The parachute is more than capable of carrying an 80,000 pound load under its massive 150 foot canopy, with assistance from exploding bolts attached to the fuselage of each of the rotorcraft vehicles.

The final design of the Atlas will consist of a rigid cross “I” shaped lifting section. This lifting section has been developed with a thin-walled pipe cross section to allow for less weight and distributed forces. There will be one cable running from each helicopter that will hold up the lifting system at either end. There will then be two cables from each helicopter running through the lifting system and connecting to the payload at its corners. The system will be equipped with explosive bolts and a ballistic parachute as a recovery system for a catastrophic failure.

The major change brought to the Chinook design comprised of increasing the lifting capacity of the aircraft. Team members discussed the different possibilities to which the applicable payload weight could be increased from the original 25,000 pounds of external payload. Ideas included reducing the amount of aircraft weight by removing most of the cargo bay, re-designing the rotor blades to increase lift and decrease blade weight, and altering the existing operation guidelines of the rotors to produce more thrust. Each of these designs was considered, and the best route found was to alter the blade design. This allowed for possible retrofitting of the existing helicopter design, without making extreme alterations in the rest of the design.

Alterations to the blade included replacing the existing airfoils used in the blade, as well as altering the internal structure of the blade. Replacing the airfoils increased the lifting capacity of the blade by using airfoils with a higher stall angle. This allowed for a higher initial angle of attack, and consequently a higher inflow of air, which, finally, allows for more thrust. The second blade alteration is a proposal to modify two sections of the internal structure of the blade. The first is along the airfoil portion of the blade. Because the internal composition of a blade is considered highly proprietary information, the internal design was based on previous hands-on experience with blade cross-sections. A semi-hollow structure was suggested, with a titanium slot near the leading edge, a fiberglass honeycomb structure surrounding the slot, a carbon fiber and fiberglass wrap covering both the slot and honeycomb structure, and lastly a steel trailing edge. These changes, along with a thicker chord, allowed for a drastic increase in thrusting capacity.

In addition to altering the blades, another modification to the rotor was used to decrease weight, along with mechanical complexity in the system. Team members looked at the rotor design of the RAH-66 Comanche aircraft, and its use of the composite based flex-beam system. This would allow for a low mechanical complexity, and allow for lighter materials used in rotor construction, decreasing the overall weight of the rotor design. The flex-beam system consists of a flexible fiberglass beam which is attached to the rotor blade through an elastomeric block. This setup eliminates the need for mechanical hinges, and helps decrease the weight associated with those attachments. This change in attachment style required a geometry change in the rotor hub. However, this change became relatively easy due to the simplistic design of the new rotor system, and only required slight changes in the rotor's attachment to the gearbox system of the aircraft. All of these changes decreased the overall density of the system, since the actual weight of the system only increased slightly. Its slight increase in weight, however, is greatly outweighed by the increase in thrust, a change that allows for a weight greater than the target payload to be lifted with enough excess thrust to climb at about 20 feet per second. For the Gun-Smash, to attain the extra horse power required for heavy lift, the 2 Lycoming T55-GA-712 turbo-shaft engines (used in the Chinook) were replaced by 2 Allison/Roll-Royce T406-AD-400 turbo-shaft engines. The engines are lubricated by a self-contained oil system capable of high output missions. A dual independent Full Authority Digital Engine Control regulates the engine system.

Power is transmitted from the engine transmission to the rotors via connecting shafts. To accommodate small misalignments between two hubs, as well as flexing and bending of the fuselage, the shaft assembly consists of multiple smaller shafts connected by flexible couplings. Each shaft section is 45.25 inches long, with an 8 inch outer diameter and a thickness of 0.25 inch constructed of AISI 4340 normalized steel alloy for high strength. Connecting these shaft sections are couplings that are specifically designed to provide axial movement and pivoting. Each section is joined at the ends by a Kaman KAflex flexible coupling and supported by bearing blocks. Connecting the engine and combining transmissions is one of these sections. The hub transmissions are connected via a synchronizing shaft consisting of nine sections with a total length of 34 feet.

The design philosophy of this project is simply stated as more lift. The Gun-Smash helicopter is specifically design to lift more than any other current United States Military helicopter. The transmission and engine therefore are two of the most crucially redesigned systems on the helicopter. The additional lift that can be gained from rotor redesign is rendered moot if the engine and transmission cannot properly drive the rotor. The Allison/Roll-Royce T406-AD-400 engines allow for an exceptional gain in horsepower while the transmission system must be fortified in order to handle the new horsepower. Heat is a major concern in the new transmission system. For this purpose, the new transmission has a dry sump system, continuously circulating oil from an external oil reservoir, which is air cooled. The dry sump affords the helicopter with a compact transmission, since there is no reason for a large oil sump in the system. The stress experienced by the transmission gears is also a major concern. The basic Ch-47 design uses variants of the Lycoming T-55 turbo-shaft engine, developing between 2200 and 2850 horsepower. The T406 engines allow for an increase in peak horsepower of more than 200% to 6150 horsepower. With this gain in power, the transmission gears must be strengthened accordingly. Inconel 625, a high nickel steel alloy, is a perfect material for the gear stock. This material was pioneered for extreme environments such as turbine blades and exhaust systems for helicopters. This alloy will expand very little with the thermal stresses of the high-speed transmission as well as forming a layer of passivation that will protect the gears during operation. The rotation rate of the new turbo-shaft engines is slower than the original Lycoming turbo-shafts; for this reason, the original reduction ratio of 64:1 is reduced to 56:1 for the Gun-Smash transmission. The transmission system integrates the torque from 4 turbo-shaft engines rotating at 12750 revolutions per minute to 2 rotors, each rotating at 225 revolutions per minute.

Design Philosophy

Mission Requirements

According to the document stating the regulations and guidelines for this proposal, it will be necessary for two helicopter vehicles to operate as a system and carry a payload 75% more than either individual rotorcraft could alone. This multi-craft system must be able to deliver the specified payload to a location 100 nautical miles from its starting location, while stopping to hover for a 10-minute period along the way; the two vehicles must then return to where the mission began. The load bearing mechanism that is to be designed must accommodate ISO containers (approximately 20’ and 48’, which can reach a maximum of 66,000 pounds), other military vehicles (wheeled, tracked, or otherwise), and other large machinery (such as construction vehicles).

Aircraft Configuration Study Findings

Initially, several designs were considered for possible configurations of the system. While there are quite a few successful designs that have been produced, including but not limited to tri-rotor, co-axial, intermeshing rotors, and compound helicopter methods, only two were seriously considered as logical options for this specific task: tandem and single-main-rotor-tail-rotor (SMRTR) configurations. In order to compare these different designs, code was written in MATLAB and run at various input configurations (see appendices for code). In order to verify that the results were accurate, each instance was tested using measured values of current helicopters that have been used. The Russian MI-26 HALO was used as a simulation of the SMRTR design, and the Boeing CH-47 Chinook represented a tandem option. The conclusion was reached that a tandem design would be more effective, mostly based upon power efficiency. Because the SMRTR design has a greater loss of engine power, it was not used in the final design. The Chinook was used as a foundation for the Gun-Smash design.

Detailed Design

Rotor and Hub

Because of the design specifications of the RFP, and the selection of rotorcraft used for this design, the redesign for the Chinook still includes two rotors operating in tandem. This setup is ideal because it optimizes the power used over a standard Single Main Rotor Tail Rotor (SMRTR) design, removing the need for a tail rotor, and thus the power loss associated with having one. The tandem design allows for large movement of the center of gravity, which is ideal for lifting large loads. For this rotor, we employed advanced design in blade design and attachment, using composite based flex-beam technology to keep mechanical complexity down, as well as made use of advanced airfoils to increase the lifting capacity of the aircraft.

A helicopter rotor encounters many different conditions during its operation. In hover, the rotor experiences different tangential velocities from root to tip, an airspeed which increases with increasing blade radius. On the retreating side, near the root of the blade, reversed flow forms. As airspeed increases, the area of reversed flow also increases, and this region saps lifting force from the rotor. At the blade tip, dynamic stall and transonic effects reduce the effectiveness of the blade's performance, and if the effects are great enough, can induce stall on the blades, endangering the aircraft, its pilot, and the mission the rotorcraft was designed to accomplish. Modern helicopters incorporate airfoils designed specifically for rotor blades, which are then used in combination to optimize the performance of the rotorcraft.

Currently, the CH-47 Chinook employs the use of the Boeing-Vertol VR-7 and VR-8 airfoils. The VR-7 is used for approximately 85% of the blade span, and uses its heavy camber to generate lift in areas where the blade's tangential airspeed isn't as large as the outboard 15%. A big problem faced when designing pitch links for rotors is the pitching moment associated with the pressure distribution along the blade when in flight. In order to help alleviate the pitching moment, as well as increase the performance of the blade, the redesign makes use of two other airfoils from the VR series, the VR-12 and 14 designs. The inboard 85% of the blade uses the VR-12 airfoil. However, the inboard 35% of the blade is also fitted with a tab along the trailing edge of the airfoil. The tab is angled 3° up, enough to reduce the pitching moment of the blade, as well as increasing the maximum performance of the airfoil at high speed and high angles of attack.

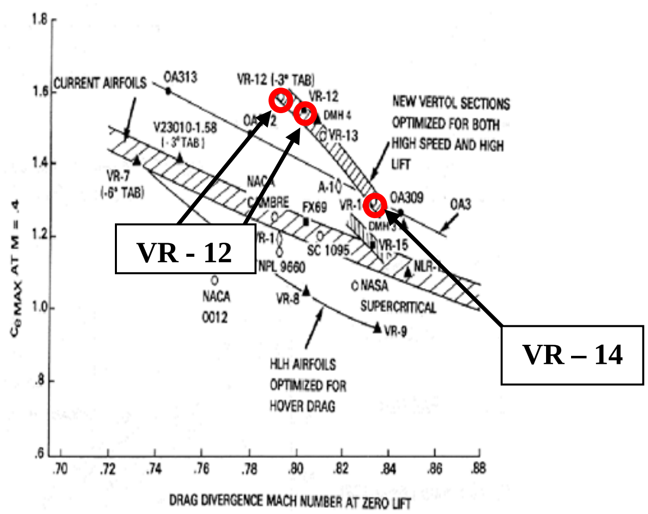

Figure 1: Rotorcraft Airfoil Performance Comparison

The VR-12 was used without the tab in the center 50% of the blade, because of its high stall angle and high lift performance. Performance characteristics can be seen in Figure 1 above. This design allows for large amounts of lift, a high divergence Mach number, and low drag values. A good characteristic that all of these airfoils have is their performance near stall. All three of these airfoils have a gradual stall behavior, a characteristic which will help curb divergent stall in implementation.

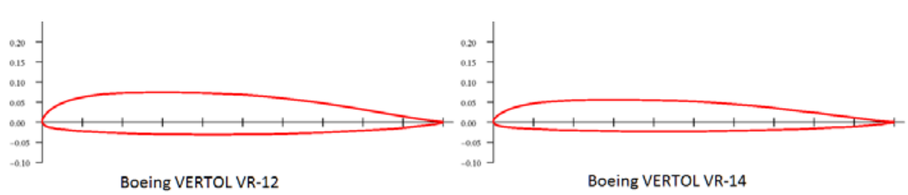

Figure 2: Cross-Sectional Shapes of Airfoils Used in Design

In order to check against existing design, computer modeling through MATLAB code was performed. Compared to the original Chinook design, airfoil performance increased significantly. After it was decided to update the airfoil geometry, further steps to increase the rotors performance were considered.

The solidity of a rotor is a ratio of the sum of the span of each blade in a rotor, divided by the rotor disc area. The solidity can be affected by a number of different criteria, including blade chord, rotor diameter, and the number of blades. A good rotor design employs a solidity that maximizes the thrust created by the rotor. In order to optimize the rotor setup, an iterative process in MATLAB was used to determine the design which would produce the most thrust for the least amount of input power. The first and largest constraint to the design was the rotor diameter. To keep the fuselage at its current dimensions, the rotor's diameter would have to stay constant at 60 feet. This left the remaining constraints of blade chord and number of blades. In order to keep blade moment down, the blade chord could not exceed a certain width. To keep the lead-lag moment down, the blade chord was increased from 2.5 to 3.5 feet. This increased the modeled performance significantly. One of the final criteria considered was the number of blades used per rotor. Each blade added would increase the solidity greatly. Before quantitative analysis was performed, it was speculated that 4 blades would be the optimal amount of blades per rotor, which would maximize the performance. At the beginning of the analysis, the number of blades was preset to 4. As analysis progressed, it seemed that the power provided by the 4 Allison/Roll-Royce T406-AD-400 turbo-shaft engines would not be enough to reach the goal thrust. However, during iteration, it was surprising to find that reducing the number of blades from 4 to 3 not only did the setup create more than enough thrust to lift the maximum payload as described by the RFP, the power required for such thrust dropped drastically, making the goal payload a definite possibility.

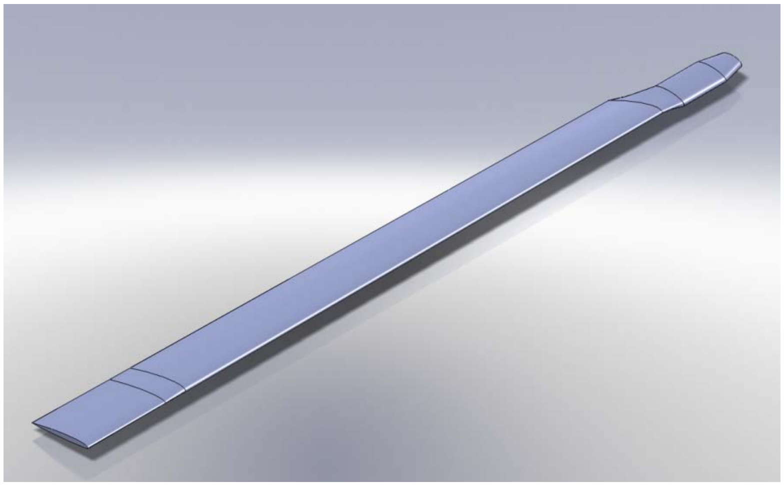

Figure 3: Prototype Blade Design

The final design consideration taken in during numeric modeling was the amount of blade twist. Span-wise twist allows for a relatively consistent amount of air flowing into the rotor disc, which allows for a constant amount of lift to be generated per unit span. The maximum angle that the blade is set at could not exceed each airfoil's stall angle, after downwash due to rotor inflow is taken into account. The final angles for the maximum helicopter climb were well below the stall angles after downwash, while still optimized for maximum lift versus power required. It was found that at a maximum climb of 6 meters per second, or 19.685 feet per second, the blades were given 26° of twist from root to tip, with a root angle of 32° twisting outboard to 6° at the tip.

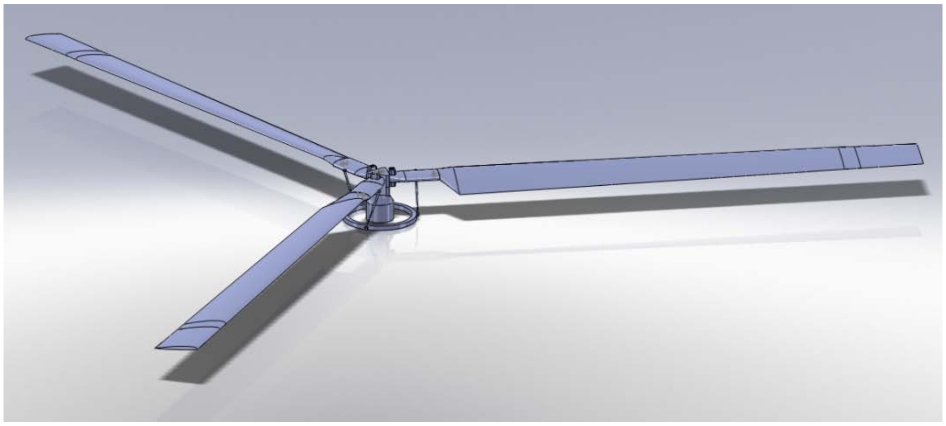

Figure 4: Assembled Rotor with 3 Blades

Because the rotor design incorporates flex-beam technology, the connection from the blade to the rotor hub differs quite greatly from the standard mechanical design of common rotors. The root of the blade is hollow, allowing for the flex-beam to be inserted inside of the blade. This cuff is needed for the deflection of the blade, almost independently of the flex-beam. This was allowed for though the implementation of a slot at the root of the blade, with an elastomeric cone to move inside the slot. This cone is attached to the beam to keep the blade from coming off of the flex-beam. Further down the cuff, another block of material (for example, a high-density rubber) is rigidly connected to the cuff on one side, and to the flex-beam on the other. This allows for some movement of the blade, including pitch adjustment, without deflecting the flex-beam too much.

As it stands, the Chinook blades are already heavy to the point that current designs employ anti-droop mechanisms in the rotor hub. A solution to this problem is to change the design of the interior of the blade, along with the materials used for the exterior of the blade to make it lighter, while at the same time retaining the structural rigidity necessary for it to operate properly. A solution to this problem would be to create a semi-hollow structure for the blade, similar to designs from Sikorsky Aircraft. Sikorsky uses a hollow metal slot along the leading edge of the rotor, with a vertically positioned fiberglass honeycomb reaching from one side of the slot to the trailing edge of the blade. This assembly is then wrapped in fiberglass, and a metal strip is used at the trailing edge of each blade to reduce damage of the thin trailing edge. This design allows for a very high rigidity while simultaneously keeping the blades extremely light.

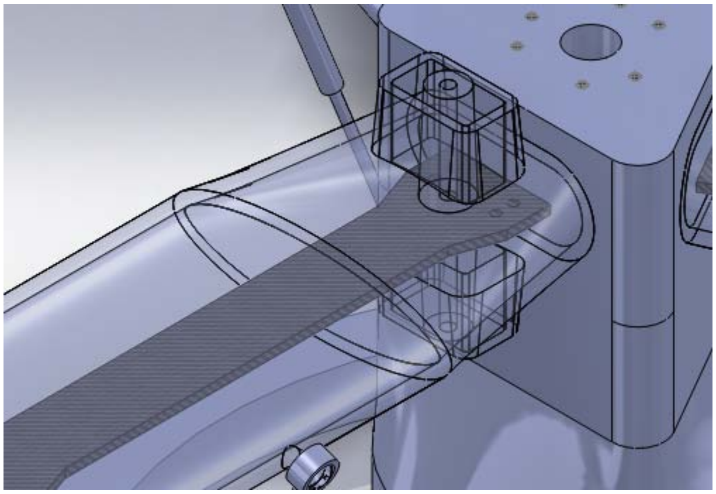

Figure 5: Close-up of Slot, Cuff, and Elastomeric Cone Coupling on Rotor Blade

While this new design would employ the latest technologies and increase the performance of the rotor, the practical design, construction, and manufacturing of these blades would be an extremely expensive process. It would cost thousands of dollars to create the proper manufacturing tools, and would cost a few thousand dollars per helicopter to build the rotors after the manufacturing process has been setup.

Engine and Transmission

The need for transmitting torque between the two transmissions required a shaft specially designed to accommodate small misalignments between two hubs as well as flexing and bending of the fuselage. First, the selection of material for the rotating shaft was achieved by identifying potential failure modes.

These failure modes arose from the stresses that the shaft would encounter during its operational lifetime. These stresses include cyclic bending stresses from transverse loads from gears, sprockets, and bearings that are mounted upon the shaft, axial stresses from helical gears or preloaded bearings as well as bending moments that sometimes fluctuate, and finally fluctuating torsion shearing stresses from the transmitted shaft torques. From these observations, it is clear that fatigue is a very important potential failure mode for power transmission shafting.

Furthermore, excessive misalignments in gear meshes, bearings, cams sprockets, or seals may lead to failure of these elements. Bending deflections or shaft slopes that lead to excessive misalignment may be said to induce failure by force-induced elastic deformation. Because the shaft is part of a dynamic system of interacting masses, it is important to examine the possibility that operation at certain critical speeds may excite intolerable vibrations. If not adequately dampened, vibration amplitudes may suddenly increase and destroy the system.

Recognizing the potential failure modes, candidate materials for power transmission shafting typically should have good strength (especially fatigue strength), high stiffness, and low cost. Steel materials meet the strength, stiffness, and cost criteria.

Most power transmission shafting is made of low- or medium-carbon steel, either hot-rolled or cold-drawn. However, if higher strength is required, such as the case for a helicopter, where the size of the shaft affects on drag and weight of the system are critical, low-alloy steels such as AISI 4140, 4340, or 8640 may be selected, using appropriate heat treatment to achieve the desired properties. With these properties in mind, AISI 4340 normalized steel was selected because of its excellent material properties, a table of which can be seen in Figure 6.

Material Property:	**Value:**
Elastic Modulus	2.05E+11 N/m^2
Poisson's ratio	0.32
Shear Modulus	8.00E+10 N/m^2
Thermal Expansion Coefficient	1.23E-05
Density	7850 kg/m^3
Thermal Conductivity	44.5 W/mK
Specific Heat	475 J/kgK
Tensile Strength	1.11E+09 N/m^2
Yield Strength	7.10E+08 N/m^2

Figure 6: Material Properties for AISI 4340 Steel (normalized)

Normalized AISI 4340's exceptionally high elastic modulus, yield strength, and shear modulus, combined with its relatively easy fabrication, make it an ideal choice for this application. Additionally, its low thermal expansion coefficient makes it perfect for even the most heavily worked shafts because it will minimize force-induced elastic deformation from friction between the bearings and the shaft material. These properties ensure that the shaft will have a minimal cross section (saving weight), while still being highly reliable over the long lifetime of the system.

Although the material selection is completed, there are still design considerations to account for, including the size requirements, vibration concerns, and noise dampening. The distance between the two transmissions is approximately 34 feet. To minimize bending stresses and dampen vibrations, this distance was divided up into nine shaft subsections of manageable 45 inch lengths connected by KAMAN KAflex mechanical drive couplings, which can be seen in Figure 7.

Figure 7: KAMAN KAflex Mechanical Drive Coupling

Helicopter flight maneuvers generate high misalignment between the engine and the transmission, or in the case of the Chinook, between the two transmissions along the top of the fuselage, which must be accommodated by the connecting driveshaft. Such driveshaft components often incorporate grease lubrication and seals. Designs of this type are susceptible to loss of lubrication, which results in overheating and possible failure, a major safety concern.

The KAflex driveshaft is a mechanical drive coupling that requires no lubrication or seals and transmits power while accommodating misalignment and length change through the use of flexible rectangular frames. A fail-safe feature enables the coupling to continue to transmit power even in the unlikely event of a failure in a load carrying member.

With the proper material and coupling chosen, finally the dimension of the wall thickness is necessary. Using the design equations (shown in the appendix), it was decided that an 8 inch outer diameter and a 0.25 inch wall thickness had an optimal weight vs. cross sectional area. A cross sectional engineering drawing of one of the links can be seen in Figure 8. In Figure 8, the ends are shown as universal joints (for illustrative purposes only); these would be replaced with KAMAN KAflex couplings as stated above. The entire shaft assembly can be seen in Figure 9.

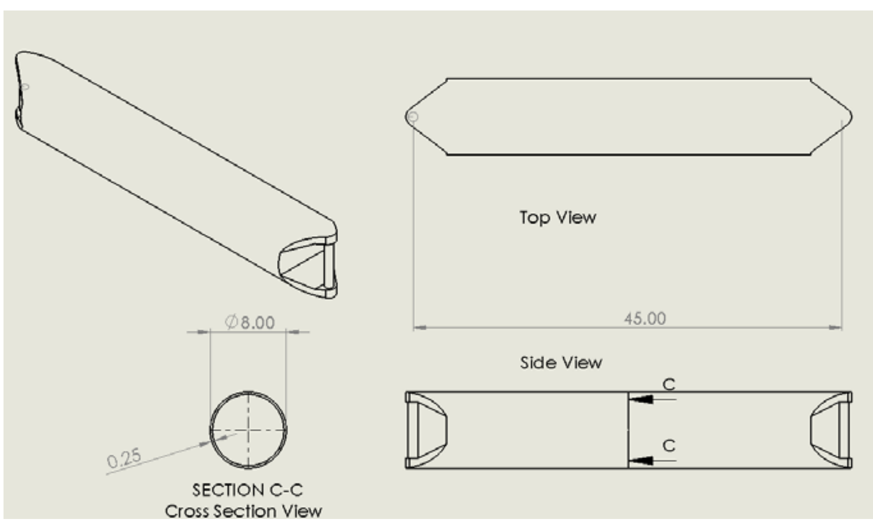

Figure 8: Single Shaft Section (Units in Inches)

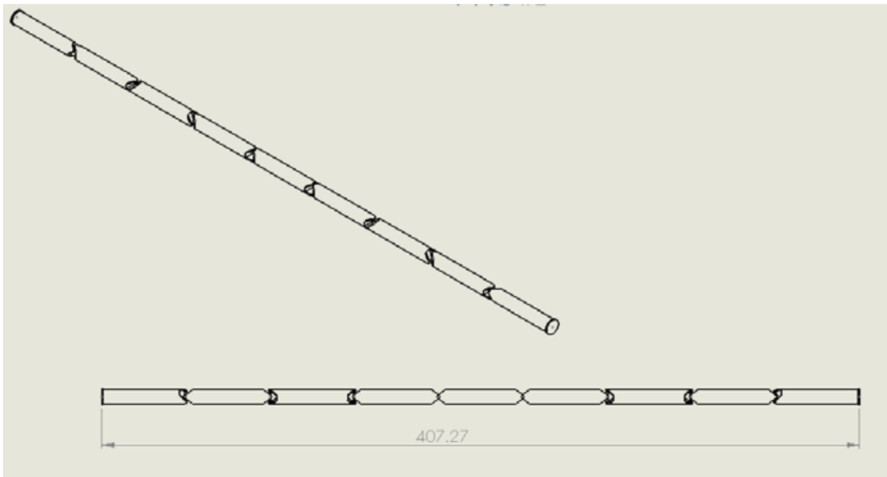

Figure 9: Full Shaft Assembly (Units in Inches)

The CH-47 is powered by 2 Lycoming T55-GA-712 turbo-shaft engines, which each produce 4,867 peak shaft horsepower and weigh 831lbs. These were replaced by 2 Allison/Roll-Royce T406-AD-400 turbo-shaft engines, which are normally used in the V-22 Osprey. Since the design required quite powerful engines, the Allison engine was chosen due to its horsepower to weight ratio, which is unsurpassed within its class. The Allison engines have a peak horsepower of 6150 at 15,000 revolutions per minute and 4,326 maximum continuous shaft horsepower at 12,750 with a dry weight of 971 pounds. As can be concluded, there is almost a 26% increase in shaft horsepower with only a 17% increase in weight. The specific fuel consumption (SFC) at maximum continuous power is 0.42 pounds per horsepower per shaft horsepower. The Allison engines are 77.0 inches long and 26.4 inches wide. Each engine is mounted on either side of the helicopter just above the fuselage, slightly forward of the rear rotor hub.

The engine oil system is of very high importance when it comes to the reliability and safety of the aircraft. It includes a tank, pump, cooler, particle detector, redundant filtration and separator for each engine. Due to the high output the engine may be subjected to, and the types of missions it will be used for, the oil capacity is kept to 4 gallons per engine.

The engine is regulated by dual independent Full Authority Digital Engine Control (FADEC). The FADEC is incorporated in the flight control systems and regulates the engine settings and adds safety in many different areas.

The redesign for the Gun-Smash concentrated heavily on the transmission. With a helicopter capable of lifting such a large load, the torque transmission system needed to be modified accordingly. With the addition of two engines, the transmission now consists of 4 engine transmissions, each connected to a Sprag Clutch assembly. From the Sprag Clutch assembly, the torque is fed into combining transmissions at either hub of the helicopter. The combining transmission then feeds the hub transmission to drive the rotor. The two transmission assemblies are connected using 9 links, designed using the KAMAN KAflex link as a model. The Sprag clutch assembly acts as a free wheel, capable of allowing the rotor hub to rotate faster than the speed of rotation of the engine. This device allows for autorotation of the helicopter. Both the forward and aft transmissions are symmetric in design, mirrored about the midpoint of the craft. This was a design choice to balance the weight distribution of the aircraft. The transmission casing is made from lightweight cast aluminum to allow for heat dissipation and a cost savings. The figure below depicts the fore transmission of the Gun-Smash.

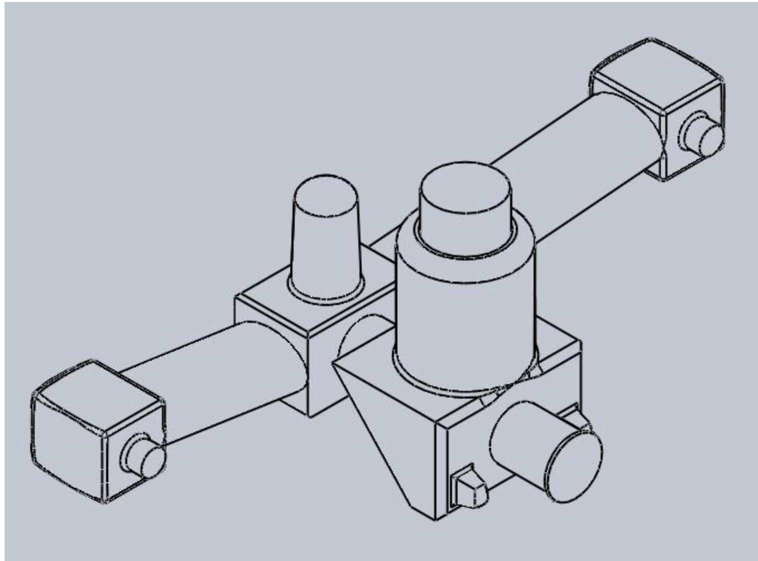

Figure 10: Fore Transmission of Gun-Smash

The gears of the transmission are manufactured from Inconel 625. Inconel is a super alloy steel, high in nickel and chromium content. Inconel is resistant to both oxidation and corrosion and is perfectly suited for the heat that the transmission gears will be exposed to. When subjected to a heat stress, the alloy will form a stable layer of passivated material on the surface, protecting it during operation. The strength of the alloy is also uncompromised as a result of the temperature stress. Inconel is also resistant to creep brought on by thermally induced crystal vacancies; aluminum and steel would both be subject to this phenomenon. The disadvantage of the Inconel alloy is the cost to machine and the weight. The Inconel super alloy is difficult to manufacture according to traditional manufacturing techniques as it work hardens very rapidly. Plastic deformation is usually a dominant mode during manufacturing, causing deformation of the part or of the tool on subsequent machining passes. This forces aggressive, but slow cutting techniques to be employed with extremely hard tools. The alloy in question lends itself perfectly to use in the transmission system because of the above highlighted positive factors including the passivation and low thermal expansion while the disadvantages are far outweighed in terms of benefit gained.

With the Inconel alloy being selected as the material for the gearing, the oil system could be designed accordingly. The transmission for the Gun-Smash has the ability to run without oil circulation for a period of 30 minutes in an effort to make a safe landing. The functioning oil system draws inspiration from high performance engines in the auto industry. The lubrication system, called a dry sump system, is gravity fed with a large external oil supply. With the oil supply being fed in from an external tank, the oil has a chance to cool and be slowly fed back into the transmission. The oil pan of the actual transmission will contain a minimal amount of oil to reduce oil movement during flight, keeping the components properly lubricated. The main oil container is mounted on top of the main transmission housing surrounding the main rotor shaft and is the pumped into each component casing with a master circulation pump. There is then an oil return on the underside of the transmission housing to an oil filtration system that removes any particles from the oil before it is pumped back into the main holding tank.

The transmission system transmits the torque from the engine through an engine transmission, which translates the torque 90° and reduces the rotation by a reduction of 1.55:1. The engine transmission houses the Sprag clutch assembly in-line with the torque shaft, allowing for the autorotation of the craft. The engine transmission then connects to the combining transmission with a shielded coupling, through the transmission nacelle. The combining transmission balances the torques received by both engines and transmits the torque to the main hub transmission with a reduction ratio of 1.825:1. The combining transmission input torque through a set of spiral bevel pinions. These pinions drive an idler gear connected to the main synchroshaft between the two hub transmissions. The synchroshaft is composed of 9 Gun-Smash coupling links. From the synchroshaft, the torque is fed into the main hub transmission with another spiral bevel pinion. This pinion then drives a bevel gear with its shaft rotating on an axis normal to the input, translating the torque for the final time, for hub rotation. As the spiral bevel rotates, it drives a primary sun gear. The sun gear drives the primary planet gears, which mesh with a stationary ring gear. As the planet gears revolve around the sun gear, the primary planet gear carrier forms a part of the secondary sun gear. This revolving sun gear drives a secondary set of planetary gears. The carrier for the secondary planet gears is integrated into the rotor shaft and is externally splined at the hub head. The reduction ratio in the hub transmission is 20:1. The overall reduction ratio of the Gun-Smash transmission is 56:1.

Fuselage and Landing Gear

An initial proposed change to the fuselage of the improved CH-47 design was that of modeling it after the Sikorsky S-64 Skycrane. The vast majority of the hull behind the cabin would be removed to make the craft lighter and therefore be able to lift more. Upon further consideration, the group came to the realization that this was not a feasible design change due to the fact that it would drastically reduce the craft's utility. It would not be able to carry any internal loads in the case that it is not carrying something under it suspended from the cargo hooks. Making the helicopter able to perform heavy lift with the system designed as well as keeping its initial capabilities was a concept that the group wanted to work towards.

No major changes were made to the actual fuselage of the improved design. It is likely that the structure would require some reinforcing in order to lift and increased internal payload but due to the uncertainty of the fuselage's construction and material make up, these changes would simply have been speculation. Ergo the main changes in regard to the fuselage were strictly external. The fore and aft cargo hooks were upgraded to match the middle cargo hook, allowing sufficient carry capacity at those locations.

With regards to the landing gear, no changes were made here either. The major reason for leaving the landing gear as they are now is that the helicopter was redesigned with the specific goal of carrying a larger external load only. Therefore, for any given scenario of internal loading, the helicopter itself will be no heavier than it would be under any present circumstance so the current landing gear will suffice for the improved design.

Load Bearing Mechanism

There were many options to consider when developing a loading system that would be used to connect two helicopters in a joint lifting mission. While brainstorming ways to lift a payload with two helicopters in a coordinated fashion, two main ideas arose. The first idea was to create an elliptical cross sectional steel ring. The idea behind the ring was that it was able to balance forces in any direction and therefore would have no moment created by the force of the payload. It was important for the cross section to be an ellipse in order to cut down on the drag forces that the ring would experience due to accelerated flight through the air. When testing this concept, weight was not an issue and therefore the ring performed quite well, with a factor of safety of around thirty – regardless of how much material was taken off during optimization while still keeping enough integrity to hold the payload. When weight was taken into consideration, it was noted that the ring weighed far more that the amount that it had been allotted for this project. The ring at its final stages weighed around 50,000 pounds, which is more than five times what the system needed to weigh in order to make the weight qualifications.

The second main idea that was brought up during the development stages was the idea of a rigid two beam cross with connection points to the helicopters at all four ends and then connection points to the payload from the underside at those same ends. The rigid cross was seemingly just as strong as the ring design but at far less of a sacrifice for weight. It was found that since the connections were at four distinct points and not evenly distributed, the ring didn't help at all with the dissipation of these forces. The rigid cross weighed around 10,000 pounds during initial development with room to decrease with further optimization of the cross section and helicopter separation distances.

An important factor in deciding overall weight and shape of the tandem system loading device is the idea of a cable attached versus a cable stayed system. The original concept when developing these models was that there would be cables running form the helicopters to the rigid lifting structure, and then from the rigid lifting structure to the payload. This concept, while the most obvious choice, is not the best way to save on materials and weight of the system. Overall, the idea is to lift a payload, which means that one would want to spend as little useable power on lifting the system that will make that possible. Attachment of the cables to the lifting system required the respective lifting system to be far more robust because it would be responsible for holding up the entire load of the payload that is in tow. This limits the amount of optimization that can be done on the final design and kept the weight very high to balance out the extreme forces of a 70,000 pound payload.

In order to counteract this fact, it was decided that the system would use a cable stayed concept in order to pass no weight of the payload on to the lifting system. This enables the lifting system to be very light because it only needs to withstand the compressive buckling forces of the cables, since they would carry the entire burden of the payload. While this requires more robust cabling than the attachment system, the reduction in weight of the whole system far outweighs that minute detail. In the new system, the cables come down from the helicopters and pass through a hole in the lifting system and go directly to the payload. The lifting system is directly attached to the helicopters by another cable on each side in order to keep it from sliding down the cables and into the payload during transfer. At this point the design is now “I” shaped in order to accommodate the pass through at the four tips of the “I” and the helicopter connections along the main axis directly in between the pass through connections. The cross section of each of these designs at this point was a standard I-beam shape in order to combat buckling. Since the device will be carrying an important load during each mission it is in service for, it has been named, “The Atlas.”

Due to the inception of the cable stay system for payload connection, a few different cross sections were analyzed to find out how material could be reduced as much as possible without sacrificing safety along the way. The initial cross section was devised as an I-beam due to its strength and overall lightweight nature. The I-beam cross section worked quite well against the buckling forces that were being exerted on the structure due to the compressive force of the cables. The problem that arose with the I-beam is that it is stronger in one direction than the other. An I-beam is made up of the flat top parts which are called the flanges and the center structure known as the web. Any forces that are in-plane with the web will be resisted by the I-beam and there should be minimal deflection. The problem with this is that the forces on the Atlas are not only acting in one plane and the I-beam lacks the same amount of strength in the both force planes. Another key characteristic is aerodynamic properties which must be taken into account when estimating power and fuel consumption. An I-beam is made up of all flat surfaces and therefore moves through the wind in a very blunt fashion. This causes an increased amount of drag, forcing the payload to be much harder to move over long distances, resulting in increased power and fuel consumption.

This initiated the concept of using a thin-walled steel pipe to create the lifting geometry needed for the tandem rotorcraft system. The thin-walled pipe, while made of steel, is very light and therefore perfect for the needs of the Atlas system. There are also benefits in aerodynamics included in the shape of the structure. In every direction of movement, there is a semi-circular shape that is far more aerodynamic than a flat plate and will save large amounts of power and fuel over great distances. Using a geometry that flows through the air more smoothly also allows the cables to be smaller and lighter because the payload will not be stressing the cables from its increased drag; this is beneficial in saving on overall weight of the system. The thin-walled pipe is also able to distribute buckling forces evenly across its entire cross section. This design makes it ideal for this scenario in which there are potentially forces in all three planes of motion.

The cable selection for the Atlas was very important and has changed throughout the design phases. Originally the cable that was selected was very thick and able to carry more than double what was needed in order to appeal to a high factor of safety. As it became apparent that weight was going to be an issue in the design of the tandem system, the cable needed to be reviewed in more detail to remove every little amount of extra weight. Once the cable stayed idea came in assumption, it was possible to greatly reduce the diameter of the cables to a 1.125 inch diameter. This cable is capable of carrying the entire payload when distributed amongst four connection points. It was also important that one cable be able to carry the entire lifting structure; this was to make sure that if one of the two cables responsible for holding up the structure fails, the other could still maintain control over the lifting structure to keep it from plummeting to the ground.

Something that was very important about the Atlas was that it was very light. It is necessary for the structure to weight no more than 25% of the overall thrust that the rotorcrafts were able to output. This would enable the system to be utilized for its intended job of lifting a payload that is 175% that of what a single Chinook helicopter can lift. The total system currently weighs approximately 10,500 pounds including the optional recovery parachute and approximately 8,500 pounds without the chute. This means that the system is only using about 15% of the total thrust in order to be lifted at its heaviest point. By making the Atlas so light, more of the thrust is able to be diverted to the most important part of the whole system - lifting an extremely heavy payload. This larger thrust offset will also allow for the system to have greater climb and cruise speeds, which allow the payload to arrive at its destination sooner. This also means benefits for the engines, since they will be operating at a good deal less than their maximum output the majority of the time; maintenance costs should be much less than if the engines were constantly put under stress.

All of the systems and modifications implemented during the design phase were put in place to uphold meticulous safety regulations. The two Gun-Smash helicopters have been attached to the Atlas at a vertical and horizontal distance that makes it impossible for the two crafts to collide. Another safety precaution that was implemented was the idea of exploding bolts at the attachments to the helicopters. This is advantageous if one of the helicopters experiences a catastrophic failure and the mission has to be aborted. All cables will release from the failed helicopter and the cable stay attachments will release form the working helicopter. This will allow the payload to drop and the lifting system to remain attached to the remaining, functioning helicopter. The cables that attach the lifting system to the helicopter have been designed to be strong enough to individually hold up the lifting system if this sort of failure occurred. Once the payload drops away, a ballistic recovery chute will deploy that is capable of lifting over 70,000 pounds. The payload will then float safely to the ground to be picked up by another working system of Gun-Smash helicopters. This ballistic “super-chute” is approximately 150 feet in diameter and has been tested with payloads exceeding 72,000 pounds. This allows for even the heaviest of payloads to be carried by the two helicopter system with a recovery method in place in case of failure.

The central portion of the structure consists of a single pipe, 150 feet long with a 26 inch outer diameter and a 0.375 inch thickness. Under a compressive loading of 23,000 pounds on each side, this design was determined to have a factor of safety with respect to buckling of 3.4, which was the lowest factor of safety found throughout the system. The size of the pipe was chosen to match with Nominal Pipe Size (NPS) standards (NPS 26 ST), and its length was chosen such that the separation between the helicopters is 150 feet. This setup makes it so that, assuming the cables hold and the beam does not buckle, it is nearly impossible for the two Gun-Smash helicopters supporting the system to collide. This is because the length of each cable between the helicopter and the beam is 40 feet, which combined with the rotor radius of 30 feet, is still less than half of the 150 foot separation.

The main pipe has a 26 inch diameter cylindrical cutaway made on each end such that the top and bottom of that cutaway are tangent and coincident with the ends of the pipe. It is connected via weld to another pipe of equal diameter and thickness on each end, perpendicular to the main pipe and fitting into the cylindrical cutaways. It was decided that the cutaway would be made from the main pipe rather than from the end pipes for two reasons. The first reason is that this results in only three separate pieces of pipe needing to be welded together rather than five, thereby reducing the number of welds necessary by half. The second reason is that the primary forces acting on the structure are applied at the ends of each end pipe, so keeping them as continuous pipes rather than separate smaller pieces serves to vastly improve the safety of the system.

These end pipes have a length that is 2.5 inches longer than the separation between the fore and aft hooks on the Gun-Smash (assumed to be 200 inches, for a total length of 202.5 inches). This design takes full advantage of the triple hook system. The central hook is attached directly to the cable stay structure, while the fore and aft hooks are vertically aligned with the cable stay blocks and are connected to cables that run through those blocks and down to the payload. The top view of the full structure can be seen below in Figure 10.

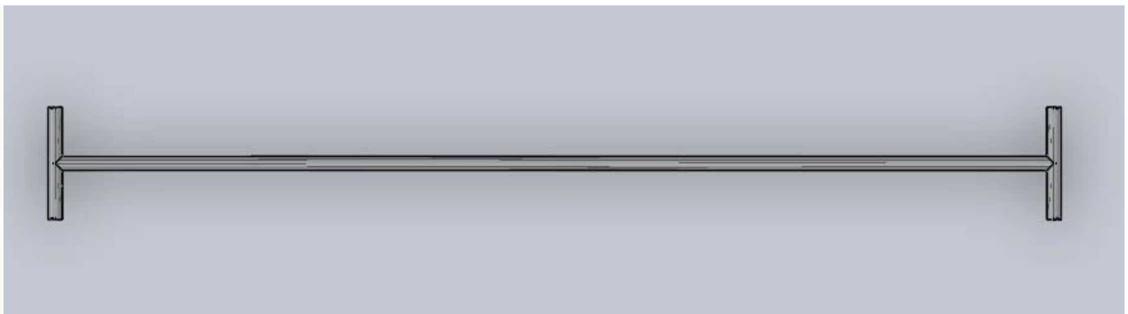

Figure 11: Cable Stay, Atlas

The cable stay piece is a solid block on the inside of each end of both pipes (for a total of four). These blocks span from the top to the bottom of the pipe and are 2.5 inches thick by 4 inches long, with their outer edges being flush with the end of the pipe. At the top, the blocks are vertical, and they experience a 30° sweep toward the center of the structure between the top and bottom of the pipe. Each block has a hole bored through it near its center.

The material used for the entire structure was Aluminum 7050-T7651. Aluminum was selected for its high strength-to-weight ratio and for its low weight-to-volume ratio because it needed to span such a great distance. This particular alloy was selected because it has a slightly higher elastic modulus that most aluminum alloys, and because it has significantly higher yield strength.

After the Atlas was designed, an attachment method from the load to the helicopter had to be chosen. A variety of methods for attachment were examined, and two systems emerged as clear leaders for the most efficient and simplest ways of attaching the load: direct attachment from helicopter to pipe and from pipe to load, or a cable stay system allowing direct connection from helicopter to load. A diagram of the connections from the helicopter to the pipe and a separate connection from the pipe to the load, as well as the resulting forces, can be seen below in Figure 11.

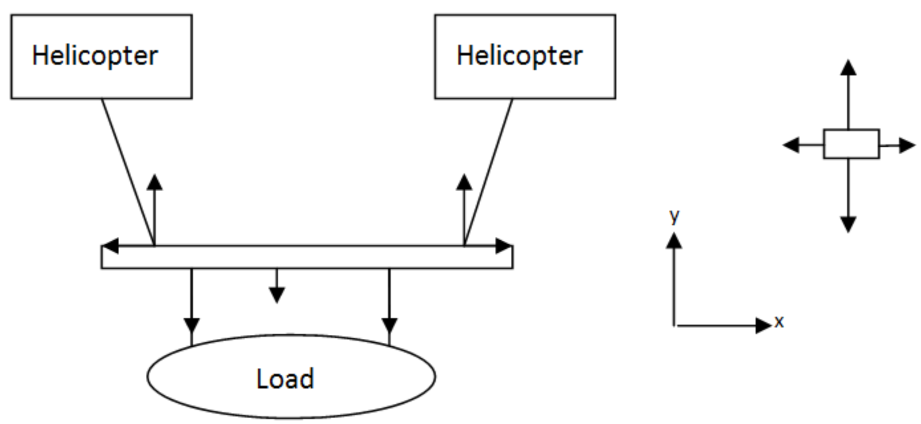

Figure 12: Connection Schematic and Free-Body Diagram for Direct Connection

As seen above, these 4 connection points cause stresses on the pipe in both the vertical plane and horizontal planes, with the larger of the forces acting across the neutral axis of the system. These large vertical forces cause sheer stress as well as a bending moment within the pipe. However, when examining the cable stay system, a different set of forces were observed acting on the pipe system. A diagram of the cable stay system and the resulting forces can be seen below in Figure 12.

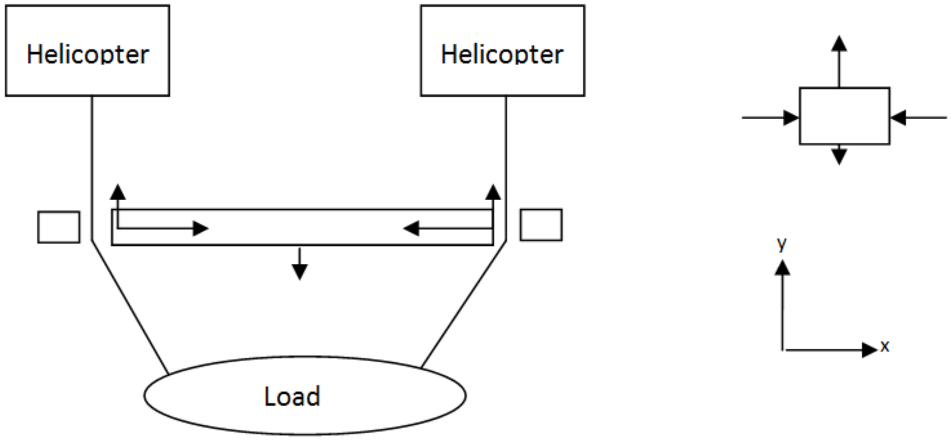

Figure 13: Connection Schematic and Free-Body Diagram for Cable Stay

Figure 12 shows that routing the cables through the pipe structure instead of attaching them to the top and bottom surfaces of the pipe cause the stresses within the structure to be in the horizontal plane of the pipe structure. The small upward force is the result of tension in the cables, but is far smaller than the weight of the structure itself. By routing the cable stays through the pipe structure, the largest stresses experienced run along the neutral axis of the pipe, as opposed to across the neutral axis as with the case of direct attachment. In order for the cable stay system to fail, the pipe would have to buckle, where as direct attachment is most likely to fail either in bending or sheer.

Comparing the two systems required analysis on the maximum stresses the pipe structure could handle before failure. Calculations show that the failure due to sheer and bending occur at significantly lower stress levels than buckling does. Because buckling is the most resilient failure mode for the pipe system, the cable stay system was deemed best and was adopted as the attachment method for the Atlas.

In order for the cable stay system to work as effectively as possible, the cables had to be routed through the pipe structure in a way that causes the least amount of stress within the system while still being easy to use. The cable stay was designed to reduce sheer forces experienced by the cable due to interaction with the pipe structure as well as to minimize the buckling forces experienced by the pipe system. The cable stay system can be seen below in Figure 13.

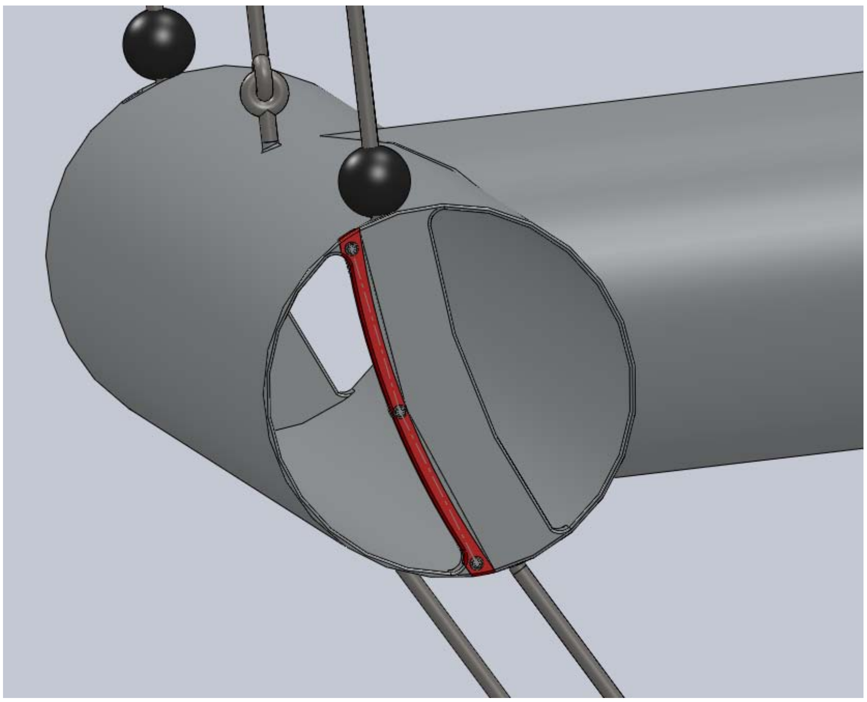

Figure 14: Cable Stay Block

As seen above, the cable stay system is a separately machined block of aluminum that is inserted into the open end of the pipe and welded into place. The block has a removable end cap (highlighted in red) that is held in place with three half-inch bolts in order to facilitate the insertion of the cables after they are attached to both the helicopter and the load. When the end cap is bolted in place, the cable rests snugly within a channel swept through a 30° arc. A 30° arc was chosen because the vertical forces on the system help to fully overcome the weight of the pipe structure while still maintaining a relatively compact system size. With the end cap in place, the cable fits snugly within the block, minimizing impacts with side walls due to vibrations within the cable caused by aerodynamic forces and load swaying. The removable caps also eliminate the need to run attachment hooks or other attachment devices through the body, minimizing the size of the hole needed, allowing the cable stay system to be smaller, saving weight and money in material. By routing the cables through a cylindrical hole, the force applied to the structure is always distributed evenly over the entire length of the swept channel. This ensures that regardless of load swaying or vibrations, the buckling force on the beam remains fairly uniform. In order to stop the cable stay system from sliding up the cables towards the helicopters, a braking system was developed.

The braking system is fairly simple; it consists of a 4 inch diameter steel ball coated with half an inch of rubber welded to each of the four main steel cables just above the location where the pipe rests in flight (seen in Figure 13 above). These balls would be in contact with the beam as the system is lifted off of the ground, and they would all be subject to approximately 800 pounds, which would increase the tension in each cable by that same amount. This braking system would have the added benefit of serving as a guideline for the crew members that are connecting the system to the payload, and because each one weighs about 10 pounds, their weight would not hinder the crew. It would allow them to place the cable at the exact location where it will be once the system becomes airborne, therefore reducing the wear on the cables by preventing the beam from sliding up them.

The cables for this system were chosen to be 1 1/8 inch steel rope for the cables attached directly to the payload (the primary cables), and 3/4 inch steel rope for the cables between the helicopters and the beam (the secondary cables). The cables were selected because the primary cables need to support a load of approximately 24,000 pounds each (80,000 pound payload plus consideration for the 30° turn and brake balls), and the secondary cables must be able to support the weight of the beam, 6915 pounds, in the event that one helicopter is forced to disconnect from the payload. The total weight of the cables for this system is 1675 pounds (including the breaks), for a total system weight of 8590 pounds.

It is also relevant to note that the Atlas can be very slightly altered to meet the RFP requirements for the original, unaltered Chinook. If the pipe used is changed from NPS 26 ST to NPS 24 SCH 10 (24 inch outer diameter with 0.25 inch thickness), the primary cables are reduced to 1 inch steel rope, and the secondary cables are reduced to 5/8 inch steel rope, then the minimum factor of safety in the system is found to be 2.59 and the total system weight is 5620 pounds, which is about 20% of the 28,000 pound lifting capacity of a single Chinook.

A vibration analysis was also done to ensure the safety of this structure. The analysis found the resonant frequency of the pipe to be 0.2 Hz, a frequency which would occur at 1.3 knots. The primary operating range of 50-100 knots creates a vortex shedding frequency range of 7.7 to 15.4 Hz, so the natural frequency of the beam is well outside of the operational frequency range. Therefore, the Atlas will not be in danger of failure due to vortex shedding.

Flight Plan

The Atlas should start out resting on a series of supports which should be placed, at minimum, with two on each end and one in the center. These supports could be designed specifically for the beam or fashioned out of wood planks, sandbags, etc., and their height should be around 2 to 3 feet for ease of cable attachment. The load to be transported should be placed from 85 to 100 feet in front of the beam, and each helicopter should land around 30 feet to the left or right of the beam. The cables, which should be on location or brought in with the helicopters, should then be attached to their respective locations, making sure that the rubber-coated steel balls on each of the primary cables are resting on the upper side of the beam. This initial configuration is shown below in Figure 14.

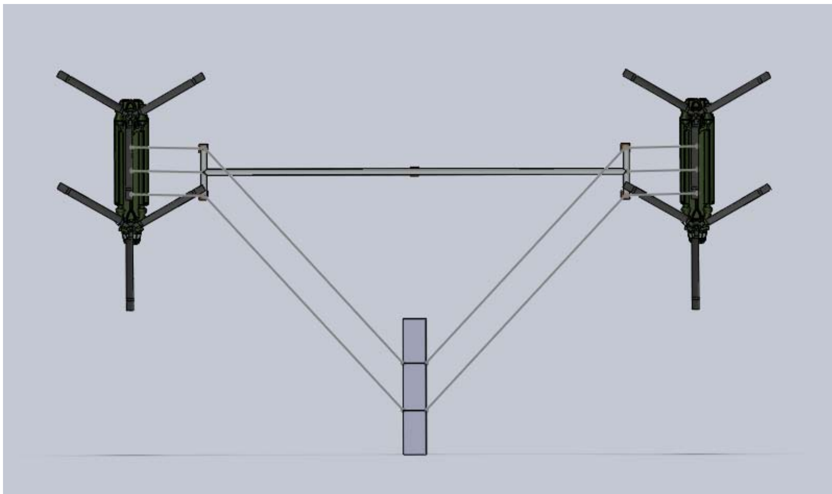

Figure 15: Pre-flight Configuration

For takeoff, one helicopter should take off slowly and hover at or just under 40 feet off of the ground, taking up the slack in the cables between the beam and the helicopters, but not lifting the beam at all. This first step is slightly complicated in that the helicopter must fly up and slightly inward at the same time, and so it may be easiest to perform in a step-like manner, first moving about 10 feet vertically, then moving in horizontally by 10 feet, and repeating that process until in position. After the first helicopter is in this position, the second helicopter should do the same. Once they are in place, both helicopters should slowly increase their altitude at the same rate until the point at which all slack in the cables is taken up. Both helicopters should then continue by simultaneously increasing collective pitch until the payload is successfully lifted from the ground. As long as this final process is done at the same rate, the setup of this system will cause the forces to balance out and induce proper alignment between the helicopters, beam and cargo before takeoff is achieved. The final flight configuration is shown in Figure 15.

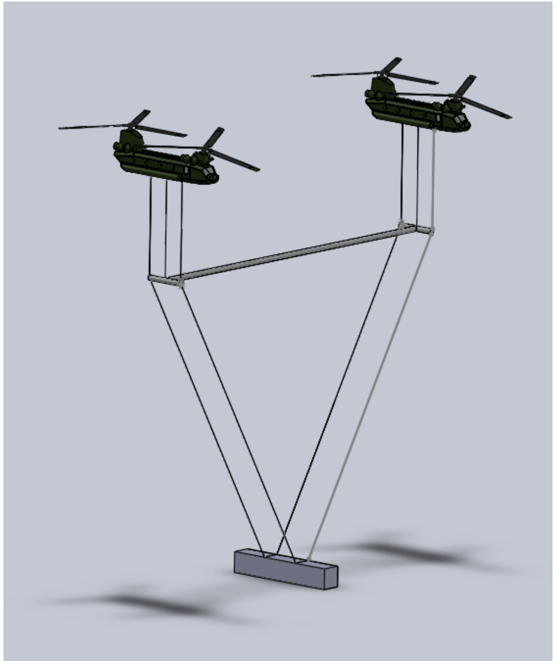

Figure 16: Flight Configuration

Once in flight, fly-by-wire controls should be engaged. This will prevent the two helicopters from putting unnecessary stresses on both the beam and their own fuselages, ensuring the safe operation of the system. The cruise speed for transportation should be between 50 and 100 knots, depending on weather conditions and the size and shape of the cargo.

Upon arriving at the destination where the cargo is to be deposited, the helicopters should carefully land the payload on the ground at the desired location and then proceed to place the beam on supports approximately 50 feet behind the load being carried. A particularly important reason for this is that it will allow the pilots on board the craft to be able to see what is going on below them pertaining to the payload. It will also permit the helicopters to remain in hover while the cables are detached from the load. The payload cables can then be relocated to the center of the beam where they will be attached to the hooks that are present there. The helicopters then take off again whilst still carrying the beam, as seen in Figure 16. Attaching the cables to hooks at the center of the beam ensures that the helicopters are not flying around with heavy cables dangling nearly 200 feet below them that could intertwine or cause potential damage to structures or people should the helicopters fly too close to the ground.

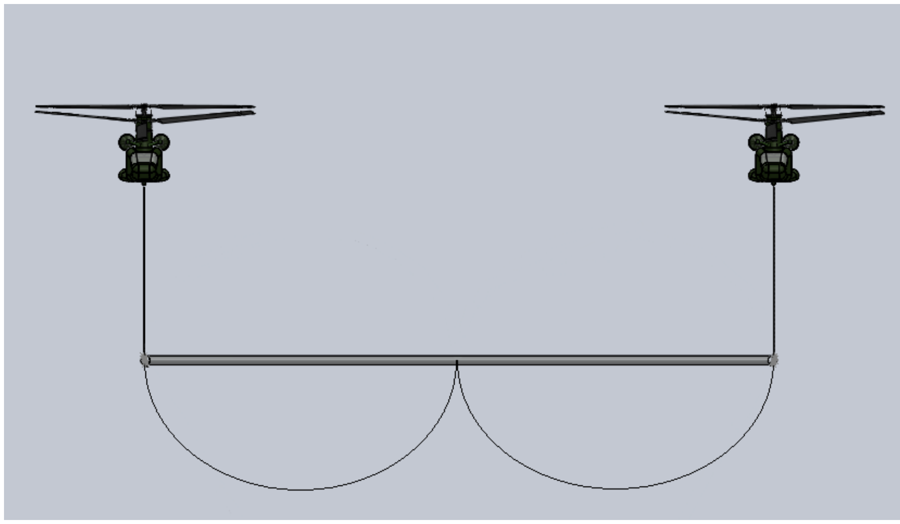

Figure 17: Return Configuration

Upon returning to the center of operation, the beam should be slowly lowered onto its original supports, at which point the two helicopters should land following an inverse procedure to that described above for takeoff. Once landed, the cables should be removed from the beam and helicopters, wound onto spools, and can be stored on location or placed inside of the Gun-Smash helicopters.

Mission Adaptability

This aircraft design can be used for various scenarios and situations where heavy lift vehicles are required. Not only is the system meant for two crafts cooperatively carrying one payload, but either helicopter has the potential to transmit a slightly lesser load than the two combined. As specified by the regulations of the Request for Proposal, the two vehicles become one for a larger task, but can be flown separately for conventional use and means.

While the cable stay system helps to prevent the collision of the two aircraft, there is still the problem of human intervention in the safety process. Any configuration involving multiple pilots in a tandem configuration, especially in close proximity such as the current design of the lifting cable stay, can lead to catastrophic failure. A solution to this problem would be the inclusion of a remotely operated ‘fly by wire’ interface, one that synchronized with another pilots controls. This would allow for one of the pilots in the configuration to control both aircraft simultaneously, with computer intervention to keep the position of the ‘unmanned’ helicopter at a constant position relative to the position of the controlling helicopter. This would allow for both of the helicopters to operate individually, as well as provide a safe way to operate both aircraft in tandem with a much smaller region of error.

An additional advantage this configuration would allow is the prospect of total remote operation, where one controller on the ground could operate both aircrafts. This would be an added safety bonus when doing heavy lift missions, as it would put no onboard lives at stake. The system would operate very similarly to the Predator and Reaper systems, with their controllers in a secluded area, operating the aircraft systems via satellite signal. This too would allow for the separate operation of each aircraft, since it would only require one additional operator to control each helicopter separately.

Summary and Conclusions

The incredible requirements of this proposal as stated by the sponsoring organization were strict yet open guidelines towards producing a rotorcraft design that could accomplish quite a feat. Many factors needed to be taken into account and modifications made to current ideas in order to successfully accomplish the goals and aspirations of this project.

To summarize this proposed design, it is crucial to take into account all suggested improvements on the Chinook to reach the Gun-Smash configuration and understand the capabilities of the Atlas. Updated high performance blades, modified with new airfoils that produced more lift, and incorporated flex-beam technology overall gave the rotor and hub subsystem a more effective output. While the new transmission weighs more than the currently used variety, it is very worthwhile due to the increased thrust it can produce. The Atlas, a pipe structure capable of integrating two helicopters into one seamless system, is a unique and original thought on the future of heavy lift rotorcraft vehicle usage. This aircraft is a reliable, efficient, and effective way to increase the payload potential of an already outstanding helicopter; it could easily become an optimal design for future missions.

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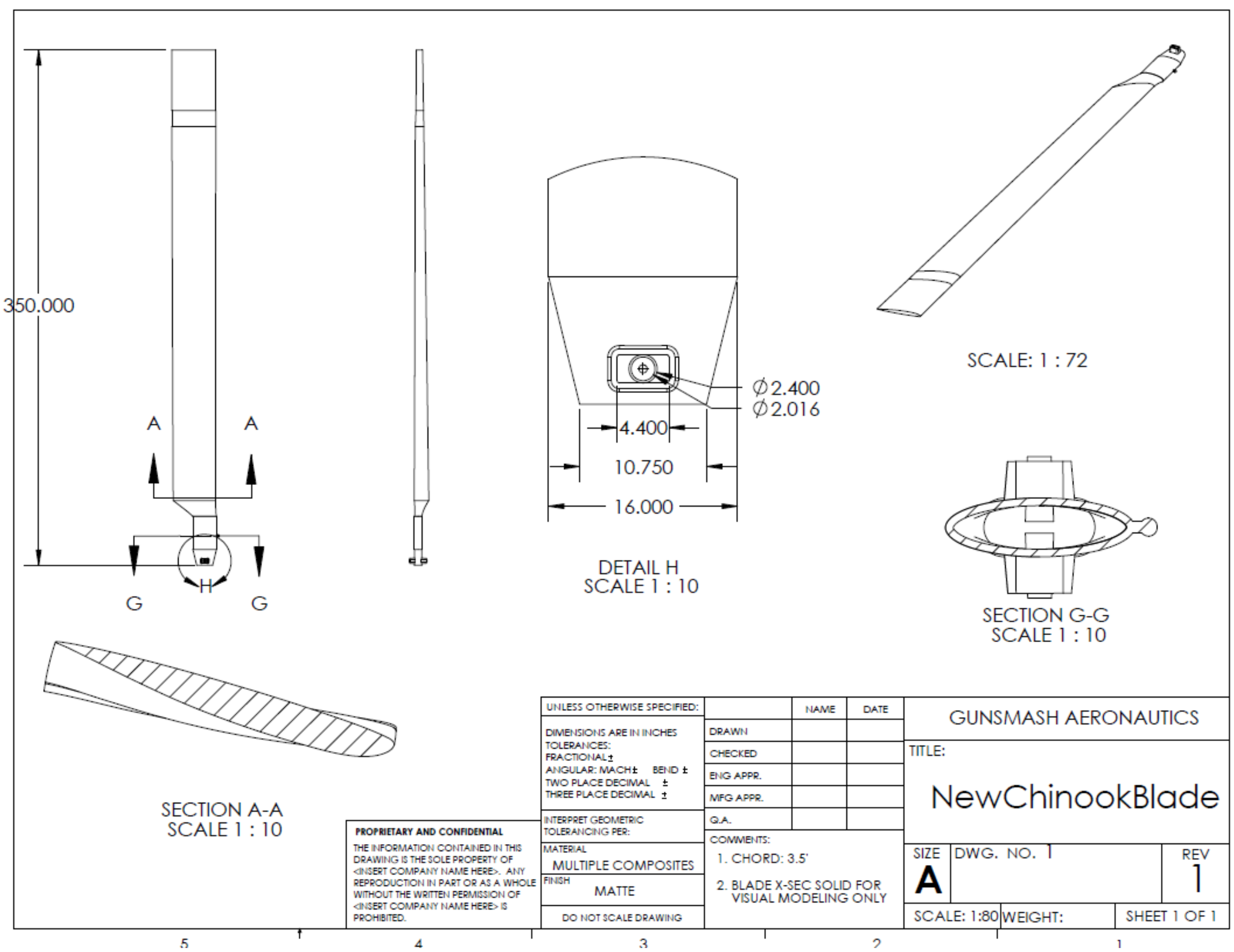
Appendices

Drawing of Re-designed Blade



Equations Used for Calculations

$$d = \sqrt[3]{\frac{32n_d}{\pi S_N}\sqrt{\left(K_{fb}M_a\right)^2 + 0.75T_m^2}}$$

$d = required\ shaft\ diameter\ for\ a\ life\ of\ N\ cycles$

$n_d = selected\ design\ safety\ factor$

$K_{fb} = fatigue\ stress\ concentration\ factor\ for\ bending\ case$

$M_a = alternating\ bending\ moment$

$T_m = steady\ torsional\ moment$

$S_N = fatigue\ strength\ for\ a\ design\ life\ of\ N\ cycles$

$$T_c = N * C_T * 1.226 * \pi * R^2 * \frac{(\Omega R)^2}{g} * 2.2$$

$T_c = thrust\ to\ climb$

$N = number\ of\ blades$

$C_T = coefficent\ of\ thrust$

$R = radius\ of\ rotor$

$\Omega = angular\ velocity$

$g = gravitational\ constant$

$$P_c = N * \left(C_P + \frac{\sigma * 0.01}{8}\right) * 1.226 * \pi * R^2 * \frac{(\Omega R)^3}{1000} * 1.34 * 1.0962$$

$P_c = power\ to\ climb$

$C_P = coefficent\ of\ power$

$\sigma = stress$

$$I = \frac{\pi\left(D_o^4 - D_i^4\right)}{64} = \frac{\pi[(26\ in)^4 - (25.25\ in)^4]}{64} = 2478\ in^4$$

$I = moment\ of\ inertia$

$D_o = outer\ diameter$

$D_i = inner\ diameter$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 * 1.04427 * 10^7 psi * 2478\ in^4}{(1800\ in)^2} = 78{,}826\ lb$$

$P_{cr} = critical\ buckling\ load$

$L = beam\ length$

$$w = \frac{Weight}{Length} = \frac{6900\ lbs}{1800\ in} = 3.833\frac{lb}{in}$$

$$\delta = \frac{5wL^4}{384EI} = \frac{5 * 3.83\frac{lb}{in} * (1800\ in)^4}{384 * 1.04427 * 10^7\ psi * 2478\ in^4} = 20.54\ in$$

$\delta = displacement\ at\ center\ of\ beam$

$w = distributed\ load$

$$k = \frac{F}{\delta} = \frac{6900}{20.54} = 336\frac{lb}{in}$$

$k = beam\ stiffness$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{336}{6900/32.2}} = 0.2\ Hz$$

$f_r = resonant\ frequency\ of\ beam$

$$St = \frac{f_r d}{U_r} \to U_r = \frac{f_r d}{St} = \frac{0.2\frac{1}{s} * 2.167\ ft}{0.198} = 2.2\frac{ft}{s}$$

$U_r = velocity\ to\ achieve\ resonant\ frequency$

$St = Strouhal\ number$

$$When\ U = 50\ knots{:}\ Re_D = \frac{UD}{\nu} = \frac{84.4\frac{ft}{s} * 2.167\ ft}{1.64 * 10^{-4}\frac{s}{ft^2}} = 1.115 * 10^6$$

$Re_D = Reynold's number$

$U = operating\ velocity$

$\upsilon = kinematic\ viscosity\ of\ air$

$$St = \frac{fD}{U} \rightarrow f = \frac{St * U_\infty}{D} = \frac{1.98 * 84.4\frac{ft}{s}}{2.167\ ft} = 7.7\ Hz$$

$$f = vortex\ shedding\ frequency$$

$$When\ U = 100\ knots: Re_D = \frac{UD}{\nu} = \frac{168.8\frac{ft}{s} * 2.167\ ft}{1.64 * 10^{-4}\frac{s}{ft^2}} = 2.23 * 10^6$$

$$f = \frac{St * U}{D} = \frac{1.98 * 168.8\frac{ft}{s}}{2.167\ ft} = 15.4\ Hz$$

*Note: Actual Strouhal Number is higher than 0.198 for these Reynolds Numbers, but specific data could not be obtained, and assuming a value for St that is lower than the true value will yield a safer result.

MATLAB Code Used for calculations

```
clear all


Nrotors=2;
Nb=3;
thetao=32*pi/180;
thetaf=6*pi/180;
dtheta=(thetaf-thetao)/85;
C=1.0668; %3.5 foot chord
R=9.144; %30 Foot radius
a=R/C;
rc = 6; %6 Meter/sec rate of climb (19.68 feet/second)
cla=7.161972/(1+(2/a));
cla2=6.611051/(1+(2/a));
sigma=Nb*C/(pi*R);

omega=(262/60)*2*pi;
cd1=0.021;
cd2=0.020;
cd3=0.0075;
dr=0.01;
ct=0.0;
cp=0.0;
r=[0.15:.01:1];

for (i=1:42) %VR-12
theta(i)=(thetao+dtheta*(i-1));
```

```
lamda(i)=((sigma*cla/16)*(sqrt(1+(32*theta(i)*r(i)/(sigma*cla)))-
1))+(rc/(omega*R));
dct=0.5*sigma*cla*(theta(i)-(lamda(i)/r(i)))*r(i)^2*dr;
alpha(i)=(theta(i)-(lamda(i)/r(i)))*180/pi;
lamda2(i)=lamda(i)/r(i);
dcp=lamda(i)*dct + (sigma*cd1/8)*dr;
ct=dct+ct;
cp=dcp+cp;
end

for (i=43:72) %VR-12
theta(i)=(thetao+dtheta*(i-1));
lamda(i)=((sigma*cla/16)*(sqrt(1+(32*theta(i)*r(i)/(sigma*cla)))-
1))+(rc/(omega*R));
dct=0.5*sigma*cla*(theta(i)-(lamda(i)/r(i)))*r(i)^2*dr;
alpha(i)=(theta(i)-(lamda(i)/r(i)))*180/pi;
lamda2(i)=lamda(i)/r(i);
dcp=lamda(i)*dct + (sigma*cd2/8)*dr;
ct=dct+ct;
cp=dcp+cp;
end

for (i=73:86) %VR-14
theta(i)=(thetao+dtheta*(i-1));
lamda(i)=((sigma*cla2/16)*(sqrt(1+(32*theta(i)*r(i)/(sigma*cla2)))-
1))+(rc/(omega*R));
dct=0.5*sigma*cla2*(theta(i)-(lamda(i)/r(i)))*r(i)^2*dr;
alpha(i)=(theta(i)-(lamda(i)/r(i)))*180/pi;
lamda2(i)=lamda(i)/r(i);
dcp=lamda(i)*dct + (sigma*cd3/8)*dr;
ct=dct+ct;
cp=dcp+cp;
end

T_climb=(Nrotors*(ct*1.226*pi*(R)^2*(omega*R)^2)/9.81)*2.2 %thrust in
pounds
T_cruise= T_climb - 26859.5

P_climb=(Nrotors*(((cp+(sigma*0.01/8))*1.226*pi*(R)^2*(omega*R)^3)/100
0)*1.34)*1.0962 %power in hp

%Drag = T_cruise/12;
%V=(P_climb/Drag)*1.94384449

plot(r,alpha)
figure(2)
plot(r,theta,r,lamda)
```